

THE ERECHTHEION AND THE LENGTH  
OF THE 'DORIC-PHEIDONIC' FOOT\*

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Why is new research needed on the length of the Erechtheion foot-standard? All scholars of the building unanimously agree that the temple was designed and executed using a foot-standard with a length of 0.326–0.328 m, and the result can be verified based on comparison of archaeological and inscriptional evidence. It is this wealth of evidence from fifth-century Acropolis which makes further scrutiny of the issue worthwhile: the building and the related inscriptions have been studied and published in detail, but a closer look at how the question of the foot-unit has been dealt with in previous scholarship shows that quite little of this material has actually been used in trying to answer the question. The method used in this paper is very different from the standard approach to architectural metrology: I find that deriving lengths of measurement-units or design principles from building dimensions is a far more complex task than is taken for granted in most earlier scholarship<sup>1</sup>, and I will argue here – using the Erechtheion as a case study – that a proper statistical analysis should be an essential part of all metrological studies.

Inscriptions such as the report by the building commission on the state of the construction work in 409/8 BC (*IG I<sup>3</sup>.474*) can direct towards studying certain elements and parts of the building, but in the end the analysis of the length of the foot-unit must be based on measurements of the actual surviving building and individual blocks. Even though in Greek metrology there is a tendency to see the

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\* I have briefly discussed the importance of the Erechtheion building block measurements in Pakkanen 2002, 501–502. The analysis given here supersedes the one presented in that paper and presents the more detailed study promised in Pakkanen 2006, 279, note 29. In addition to my home institution, Royal Holloway at the University of London, the research has been financially assisted by a grant from the Central Research Fund of the University. Jim Coulton, Richard Tomlinson and Esko Tikkala have commented on an early version of the manuscript. Needless to say, I am very grateful to all these individuals and institutions.

A *table ronde* on various methodological approaches is announced in the *Talanta* editorial preface, *supra* 7.

<sup>1</sup> For further criticism and evaluations of previous metrological studies on Greek architecture, see Pakkanen 2002; 2004a; 2004b; 2005.

finished structure being directly linked with the design of the building, what is actually being studied is the end-product which is several steps removed from the original conception. This is one reason why it is necessary to try to take into account as much of the data as possible. Using only a few dimensions to define the employed foot-unit risks being unduly influenced by arbitrary factors: it is quite conceivable that the sizes of the chosen building elements were not even designed to be a precise multiple of the employed measurement-unit or that the builders did not meticulously follow the design; possibility of modern measurement errors and the condition of the monument have to also be taken into account. Increasing the size of the data set reduces the likelihood of individual dimensions invalidating the analysis.

Even if the size of the construction foot-standard can be derived from the building measurements, this does not necessarily solve the problematic relationship between the design and the end-product. We can, however, be one step closer towards solving the controversy between the employed foot-standards and architectural design: since the analysis of the unit makes no assumption about the initial design, it can be studied whether the defined foot-unit can be related to major building dimensions in a meaningful way. Standard design analyses assume that this is the case, but it has rarely been demonstrated in the context of fifth-century Greek architecture.

Analysis of a larger data set than in previous studies necessitates the use of an appropriate quantitative method: increasing the number of dimensions results in more complexity and, therefore, it is impossible to recognise the emerging patterns employing the traditional methods of metrology. In general, I think that the question of the use and definition of measurement-units in Greek architecture is far from being solved and that the whole procedure of deriving foot-unit lengths from architectural dimensions is in need of further evaluation. The current lack of consensus among scholars could indeed be a reflection of the fact that no widely used foot-standards existed in the Greek world<sup>2</sup>, but it is also at least partially due to the use of inappropriate methodology in trying to solve the question at hand: besides being an archaeological problem, determining the length of Greek length-units from building measurements is a statistical one, and studies which do not use quantitative methods can easily reach incorrect conclusions<sup>3</sup>. Also, since earlier scholarship on foot-units has often used procedures of questionable validity, contemporary ‘understanding’ on the sizes of the measurement-standards should not be taken as a starting point of any new research. Statistics can be used to over-

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<sup>2</sup> See e.g. Coulton 1974, 62; de Waele 1988, 205–206; 1990, 1; Cooper 1996, 131–132; Wilson Jones 2000, 75.

<sup>3</sup> Pakkanen 2004a; 2004b; 2005. In general, metrological analyses of Greek architecture have not used quantitative methods to any great depth, though R. C. A. Rottländer’s work should be mentioned here as an exception; see e.g. Rottländer 1996.

come this problem: it is not necessary to make preliminary assumptions about which hypothetical unit could have been employed.

Because of the wealth of information on the Erechtheion, it is unlikely that further research would result in a radically different foot-unit: unlike with other buildings, the list of the building block sizes given in *IG I<sup>3</sup>.474* has since late nineteenth century acted as a reality check so that no unlikely results have been put forward. The reason why I have nevertheless chosen to use the Erechtheion as a case study is primarily a methodological one. Since the use of statistics in Greek metrology is not currently regarded as essential, it is vital to put forward a test case where the results of the analysis can be independently validated. This is the only way to demonstrate how critical the use of proper methodology is in the analysis of other buildings for which no cross checks are possible to carry out: in most cases metrological analysis can solely be based on archaeological material.

I will in this paper employ two different computer-intensive statistical methods. Bootstrap confidence intervals can be used in archaeological contexts to determine how precisely a certain length, such as a foot-unit, can be derived on the basis of inscriptional and architectural data<sup>4</sup>. The second, more complex quantitative method is based on D. G. Kendall's cosine quantogram analysis for detecting a quantum, or a basic dimension, of unknown length in a set of measurements; after the first stage of analysis the result validity can be evaluated through computer simulations<sup>5</sup>. Monte Carlo simulations can also be used to determine the probable range of a statistically valid quantum (Kendall 1974, 259–260; Pakkanen 2004a, 270).

I will first use cosine quantograms to analyse the dimensions of the building blocks listed in the Erechtheion inscription. Since the number of identified blocks is quite small, it is also necessary to study a larger set of measurements from the temple: I have chosen to use the plan dimensions to further scrutinise the length of the Erechtheion-foot, and at the same time some of the reasons behind the shortcomings of previous metrological analyses can be brought forward.

### **Previous scholarship on the Erechtheion-foot**

The idea that the Erechtheion foot-standard can be defined as *ca.* 0.326–0.328 m was put forward by Wilhelm Dörpfeld in the late nineteenth century. He was the first to perceive the potential of the Erechtheion block inventory in defining the length of the fifth-century Athenian foot-unit: he compares the dimensions listed

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<sup>4</sup> On bootstrap methods in archaeology, see Pakkanen 1998, 53–55; Baxter 2003, 148–153.

<sup>5</sup> Kendall 1974; for using the method in studies on Greek architecture, see Rottländer 1996; Pakkanen 2002; 2004a; 2004b; 2005. Kendall's method has been recently reviewed in Baxter 2003, 228–235.

in the inscription with building block measurements, but ends up being disappointed with the results as the correspondence between the dimensions and the written testimony is not perfect. To solve the question, Dörpfeld turns to some known buildings: he takes the interior width of the Erechtheion, the central nave and column height of the Parthenon, the radius of the orchestra of the theatre of Dionysos, the column height of the Propylaia and the interaxial distance of the interior colonnade of the stoa of Eumenes. These all seem to have been executed using a round number of feet and he proceeds to suggest that a measurement-standard of 0.326–0.328 m can therefore be recognised in Athenian architecture (Dörpfeld 1890b, 168–71). Dörpfeld’s approach is understandable: he has a specific problem for which he needs a quick solution. What is not generally recognised is that the method of data selection is clearly invalid: his choice of analysed elements is dependent on them producing a good fit with a predetermined foot-unit. There is no way of knowing that these specific parts of the buildings were originally designed *and* executed as an exact multiple of the standard in question. It is quite unfortunate that his approach has since become nearly universally accepted in studies of Greek architecture and can now even be called ‘the standard metrological method’. Despite the shortcomings of the initial analysis, Dörpfeld’s foot-unit has subsequently been regarded as one of the most widely used standards in the Greek world, and it is most often called the ‘Doric’ or ‘Pheidonic’ foot<sup>6</sup>.

The American team responsible for the publication of the 1927 monograph on the Erechtheion considered the question of the foot-unit already solved by Dörpfeld, so they did not regard that any further reflection on the issue was necessary<sup>7</sup>. William B. Dinsmoor’s original analysis of the length of the foot used on the Acropolis is hardly more thorough than Dörpfeld’s: it is based on five Erechtheion and five Propylaia dimensions<sup>8</sup>. Dinsmoor uses the height, length and thickness of the wall block and the column diameter and interaxial spacing of the North Porch to arrive at a foot length of 0.32600 m for the temple. The wall block dimensions are reported by Dinsmoor as 0.489 m × 1.304 m × 0.652 m, and his choice of these particular figures is quite interesting as they all produce a foot-unit of precisely 0.326 m<sup>9</sup>. His dimensions can be compared with the first three

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<sup>6</sup> The most often quoted analysis of the ‘Doric’ foot is Dinsmoor 1961, 358–360, but he suggested the use of the term already in Dinsmoor 1940, 20 note 40. Dörpfeld 1890b, 177 is the first modern scholar to connect the name of legendary Argive king Pheidon with the foot-unit; Herodotos 6.127 recounts that he introduced new measurement standards to Argos. For recent scholarship on the foot-standard, see Wilson Jones 2000.

<sup>7</sup> Stevens *et al.* 1927, 222–223, esp. note 3. They use a value of 0.328 m for the unit, even though Dörpfeld was not this precise in his analysis.

<sup>8</sup> Dinsmoor 1961, 358–359. The recent analysis in Dinsmoor/Dinsmoor (2004, 5–7, 447–449) is slightly more detailed, but it concentrates solely on the Propylaia.

<sup>9</sup>  $0.489 \text{ m} / 1.5' = 0.3260 \text{ m}$ ;  $1.304 \text{ m} / 4' = 0.3260 \text{ m}$ ;  $0.652 \text{ m} / 2' = 0.3260 \text{ m}$ ; Dinsmoor 1961, 358.

lines in Table 1: depending on which specific examples are analysed, it is also possible to arrive at a little more variation of 0.325–0.327 m based on the wall block sizes. His two dimensions of the North Porch give a length of 0.326–0.327 m for the unit<sup>10</sup>. Dinsmoor’s approach takes partially into account the inscriptional evidence when he uses the wall block dimensions given in *IG I<sup>3</sup>.474.11–12*, but no such justification can be given for the choice of the North Porch details.

Hansgeorg Bankel’s relatively recent analyses of the Erechtheion employ a graphic method he calls the “metrological scale” (Bankel 1983, 67–70, 89–91; Bankel 1991, 159–162). It uses seven dimensions of the North Porch in the graphic analysis and nine in the table calculations, though the precise size of his foot-unit 0.32674 m is derived from one single measurement, the stylobate width of the porch<sup>11</sup>. Dörpfeld and Dinsmoor base their analysis of dimensions which can be expressed in terms of full and half-feet, but Bankel assumes that it is justifiable to introduce dimensions as small as  $\frac{1}{32}$  of a foot or *ca.* 1 cm<sup>12</sup>. Bankel’s investigation is more complex than the two cases discussed above: even though his approach has been criticised by several scholars (Wesenberg 1984, 549–553; Büsing 1985, 159–160; Pakkanen 2005), demonstrating why it fails to correctly derive the foot-standard requires a more thorough examination of the plan dimensions. I will, therefore, need to return to the matter later on.

Rolf C. A. Rottländer is not in general convinced of the existence of the ‘Doric-Pheidonic’ foot in antiquity, so he tries to interpret the dimensions listed in the Erechtheion inscription in terms of his ‘Drusian’ foot of 0.33317 m (Rottländer 1991). His foot-standard produces systematically greater lengths than the dimensions measured on the building, so his argument can be considered rather more tendentious than persuasive.

### ***IG I<sup>3</sup>.474* and the length of the Erechtheion-foot**

The importance of the Erechtheion construction work inventory for our knowledge on the length of the foot-unit employed in fifth-century Athens and Attica

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<sup>10</sup>  $0.817 \text{ m} / 2.5' = 0.3268 \text{ m}$ ;  $3.097 \text{ m} / 9.5' = 0.3260 \text{ m}$ ; Dinsmoor 1961, 358.

<sup>11</sup> Bankel’s unit derivation is criticised in Büsing 1985, 159–160, though I do not agree that a design analysis based on the “theoretical length” of the foot-unit produces necessarily any more reliable picture of Greek architectural practices than Bankel’s method.

<sup>12</sup> Bankel 1983, 67–70, 89–91; Bankel 1991, 159–162. This aspect of Bankel’s analysis is evaluated by Burkhardt Wesenberg, and he shows that an ‘Attic’ foot of 0.29474 m can actually produce a slightly better fit with Bankel’s data set. Wesenberg also demonstrates that employing half-dactyls in metrological analyses reduces the discrepancies between foot-units and measured dimensions nearly meaningless; see Wesenberg 1984, 549–553. See also the analysis of Fig. 4 below.

cannot be overemphasised<sup>13</sup>. The inscription gives a good indication which building dimensions form the most useful starting point for a study concentrating on the relationship between architectural measurements and Greek foot-standards<sup>14</sup>, and it also provides a way of checking the validity of the proposed statistical method.

It is possible to match 21 dimensions listed in the inscription against actual measurements taken on the building blocks (Table 1)<sup>15</sup>. An estimate for the length of the Erechtheion-foot can be obtained by dividing the measured dimension by the foot-value given in the inscription. The data of the inscription cannot be regarded as entirely precise (column 4 in Table 1): the produced range for the length of the foot-unit is rather wide at 0.285–0.342 m. There is, however, no need to give up the analysis at this point, as Dörpfeld did (Dörpfeld 1890b, 168–171), but rather to proceed to see if statistical analysis can provide more information on the matter. The mean of the derived foot-lengths in column 4 of Table 1 is 0.322 m, but it gives no indication of the probable range of the standard used. For this we need to turn to computer-intensive statistics since archaeological material seldom meets the two main assumptions for using classical confidence intervals for small samples: the original ‘population’ of determined lengths should be normally distributed and the analysis based on a random sample (see e.g. Shennan 1997, 79–83). Using the calculated foot-units in Table 1, the 95% bootstrap confidence interval for the Erechtheion-foot mean length cannot be determined more precisely than as 0.316–0.327 m<sup>16</sup>. The ‘standard’ length of 0.326–0.328 m for the ‘Doric-Pheidonic’ foot, originally defined largely on the basis of few dimensions

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<sup>13</sup> *IG* II<sup>2</sup>.1668 and the discovery of the remains of Philon’s Arsenal in the Piraeus are equally important for the fourth-century, though determining the length of the foot-unit used is not entirely without controversy; for the physical remains, see Steinhauer 1994, 1996; for a discussion of the problems in identifying the range of the unit, see Rottländer 1997; Pakkanen 2002, 502–503, 505, table 4.

<sup>14</sup> G. Ferrari has recently resurrected Dörpfeld’s theory that the Old Temple of Athena was not completely pulled down after the Persian sack of the Acropolis, but was still standing when Pausanias visited the Acropolis; Dörpfeld 1887a; 1887b; 1890a; 1897; Ferrari 2002. A reinterpretation of *IG* I<sup>2</sup>.474 is at the heart of her argument, but comparison of inscriptional and archaeological evidence clearly shows that the inscription is solely related to completing the construction of the Erechtheion, not the Dörpfeld temple; see Pakkanen 2006 (for a recent article accepting the conclusions presented in this paper, see Gerding 2006, 390; Linders 2007, 781).

<sup>15</sup> A detailed analysis of the relationship between the inscription and the Erechtheion block measurements is presented in Pakkanen 2006.

<sup>16</sup> A 95% confidence interval for the unit length means that the Erechtheion foot-unit is with 95% probability within the defined range. Various bootstrap methods produce slightly differing results for the confidence interval: based on 5000 bootstrap samples the percentile method gives a range of 0.3164–0.3274 m, the bias-corrected and accelerated method (BCa) 0.3163–0.3273 m, and bootstrap-*t* (studentised) method 0.3143–0.3273 m. As expected, the overall agreement of the bootstrap methods with the classical interval of 0.3165–0.3279 m is good. The bootstrap programs used in the analysis have been implemented on top of the statistical software Survo MM; Reijo Sund has programmed the module producing the percentile

1. Block		2. <i>IG I<sup>3</sup>.474</i> (feet)	3. Measured dimension (m)	4. Length of foot unit (m)
4 wall blocks:	L (lines 10–11)	4	1.30	0.325
	W (line 11)	2	0.652	0.326
	H (lines 11–12)	1 <sup>1/2</sup>	0.490	0.327
5 epikranitis blocks:	L (lines 16–17)	4	1.301	0.325
	H (lines 17–18)	1 <sup>1/2</sup>	0.492	0.328
Corner epikranitis:	W (line 20)	4	1.242	0.311
	H (lines 20–21)	1 <sup>1/2</sup>	0.492	0.328
8 architrave blocks:	L (lines 33–4, 37–8)	8	2.608	0.326
	W (lines 34–5, 38–9)	2 <sup>1/4</sup>	0.77	0.342
	H (lines 35, 39)	2	0.63	0.315
3 Karyatid Porch roof blocks:		13	4.200	0.323
	L (lines 87–8)			
East frieze block:	W (lines 88–9)	5	1.648	0.330
	L (lines 115–19)	6 & 8	1.940 & 2.675	0.323, 0.334
	W (lines 115–27)	1	0.285, 0.315	0.285, 0.315
North frieze block:	H	2	0.617	0.309
	W (lines 115–27)	1	0.298	0.298
	H	2	0.683	0.342
Geison block:	L (lines 128–55)	4	1.301	0.325
	W	3	0.998	0.333
				$\bar{x} \approx 0.322$

Table 1. Comparison of building block dimensions in *IG I<sup>3</sup>.474* and the Erechtheion (for sources of the dimensions in column 3, see Pakkanen 2006, table 1).

in Attic buildings, is at the very end of this range (Dörpfeld 1890b, 171; Dinsmoor 1961, 358). Evidently a more thorough study is in place.

The best place to start is to take the block dimensions listed in column 3 of Table 1 and subject them to independent statistical analysis: this means that the information given in *IG I<sup>3</sup>.474* is solely used to select the analysed blocks and the data on their size in feet are disregarded at this stage. This gives a set of measurements which should have an underlying basic dimension, a foot-standard in this case,

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and BCa intervals and the author of this paper the bootstrap-*t* module. On the use of bootstrap methods in architectural contexts, see Pakkanen 1998, 53–55; 2004b, 102–103; Pfaff 2003, 84. For a recent overview of the archaeological use of bootstrap, including an evaluation of the Tegea column analysis presented in Pakkanen 1998, 53–54, see Baxter 2003, 148–153. On bootstrap methods in general, see Davison/Hinkley 1997; Manly 1997, 34–68.

which produces the observable lengths. In statistical terms this dimension is called a quantum; in the case of the Erechtheion the ‘quantum hypothesis’ is that a block dimension  $X$  can be expressed as the product of an integral multiple  $M$  times the quantum  $q$  plus an error component  $\varepsilon$ . In mathematical terms this can be denoted as

$$X = Mq + \varepsilon. \quad (1)$$

The critical factor in the formula is error  $\varepsilon$ : it sets a lower limit for quantum  $q$ . In any case  $\varepsilon$  or  $q - \varepsilon$  should be substantially smaller than any considered  $q$ <sup>17</sup>. Variation of  $\pm 0.01$  m between similar smaller architectural elements is quite typical of Greek building practice<sup>18</sup>, but by computer simulations it can be demonstrated that an error of this size has no effect on detecting a quantum in the region of *ca.* 0.08 m, or one quarter of a ‘Doric’ foot, even when the number of analysed building dimensions is small (Pakkanen 2002, 502–503). In order to give due consideration to units slightly smaller than a ‘normal’ quarter-foot or palm, I will use a range of 0.06–0.40 m in the following analyses. The upper end is chosen so that it is clearly greater than any suggested Greek foot-standard<sup>19</sup>.

In order to determine how well a block measurement  $X$  can be expressed in terms of quantum  $q$ ,  $X$  needs to be divided by  $q$  and the remainder  $\varepsilon$  analysed. The value of  $\varepsilon$  will be between 0 and  $q$ , and the less it deviates from either 0 or  $q$ , the better the fit between  $X$  and  $q$ . In Kendall’s cosine quantogram analysis  $\varepsilon$  is first divided by  $q$  and then the cosine of the quotient is taken: this gives a value of +1 for dimensions  $X$  which are an exact multiple of  $q$ , and the worst fitting measurements produce a value of –1. To find out which  $q$  is the best candidate for the quantum, it is necessary to compute the cosine value for all the measurements  $X$  and the full quantum range. How well the tested  $q$  values fit the data can be determined from the cosine quantogram where the sum of the cosine values is plotted against  $q$ : the highest observable peak in the graph is the most likely quantum candidate (see Figs. 1 and 4). All this can be expressed as the following mathematical formula for the quantum score:

$$\phi(q) = \sqrt{2/N} \sum_{i=1}^n \cos(2\pi\varepsilon_i / q). \quad (2)$$

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<sup>17</sup> Since  $\varepsilon$  as a value between 0 and  $q$ ,  $q - \varepsilon$  can also produce an error significantly smaller than  $q$ .

<sup>18</sup> Coulton 1975, 94. From a statistical point of view it does not matter whether the observed variation is due to factors in Greek building design and execution, the current condition of the blocks or modern measurement errors.

<sup>19</sup> The ‘Samian’ foot of *ca.* 0.35 m; for references to the foot, see Wilson Jones 2000, 75 n. 16. There is no need to consider longer units such as cubits in this case since based on *IG I<sup>3</sup>.474* it is known that a foot-standard rather than anything substantially longer was employed in the building construction.



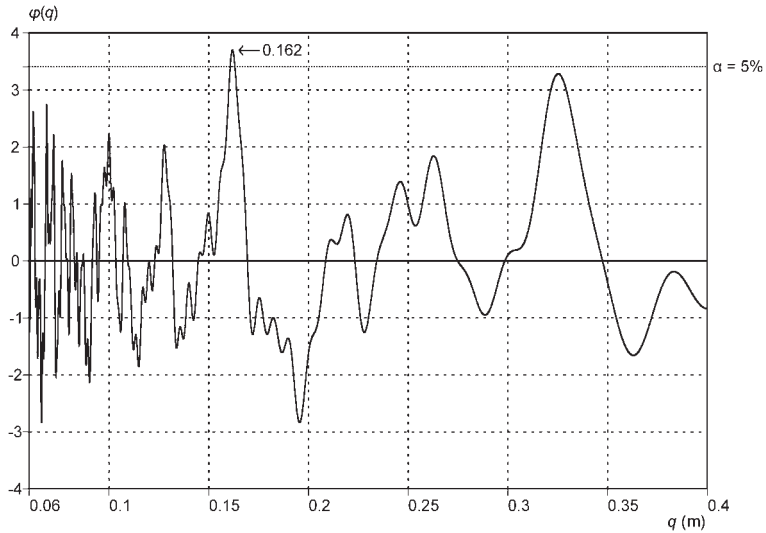


Fig. 1. Cosine quantogram of the Erechtheion building block measurements listed in Table 1.

Here  $N$  is the number of measurements and the first term  $\sqrt{2/N}$  a scaling factor: in order to avoid getting a higher value for  $\phi(q)$  by simply introducing more measurements, the cosine sum needs to be scaled down (Kendall 1974, 235–239).

Fig. 1 presents the cosine quantogram of the measurement data in column 3 of Table 1. There are two apparent peaks, the first at 0.162 m and the second almost exactly twice the first at 0.325 m; the first corresponds obviously to the half-foot of the unit employed and the second to the full foot. Since the statistical analysis makes no *a priori* assumption about the quantum size, or even its existence, it is highly significant that the cosine quantogram method points towards a slightly shorter unit than the current consensus on the length of the ‘Doric’ foot. The quantum score of the first peak is 3.70 and the second significantly less, 3.28. The next task is to find out whether the peaks are sufficiently high to be considered real quanta and not just background noise; if they are, then it would be convenient to know how precisely the length of the unit can be defined on the basis of the building block measurements.

The best means of evaluating whether the highest quantum score produced in the initial analysis is statistically significant is to build mathematical models of the data and use them to produce random non-quantal simulation data sets. These should have the same statistical properties as the original set of measurements, but lack the quantal properties. The replica data sets are then analysed in the same

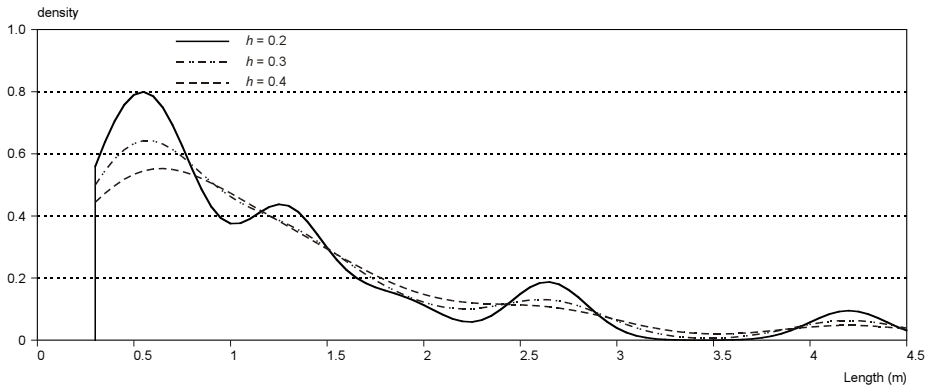


Fig. 2. Kernel density estimation distributions used to produce simulation data sets. The curves are based on the Erechtheion building block measurements listed in Table 1.

way as the primary data, and if the simulated function peaks are systematically lower than in the initial analysis, it is possible to accept the quantum hypothesis: the highest original peak can in that case be regarded as a valid candidate for the quantum<sup>20</sup> and directly related to the foot-standard used in the Erechtheion. Due to the random nature of the computer simulations, the method for testing the validity of the results is often called Monte Carlo analysis.

Using kernel density estimation (KDE) distributions is an effective way of producing the non-quantal data sets needed in the simulations<sup>21</sup>. The idea behind the KDE is that a small continuous distribution is placed at the position of each observation and these are then added together to create a smooth curve (Fig. 2). The shape of an individual ‘bump’ can be seen at the right of the figure (solid line). Employing KDE to produce distribution models emphasises the notion that the existing measurements are the most reliable guide to what the general character-

<sup>20</sup> Kendall 1974, 241–249; Fieller 1993, 282–283; Pakkanen 2002, 501–502; 2004a, 263–270; Baxter 2003, 231–233.

<sup>21</sup> On KDE distributions used as mathematical models to produce non-quantal data sets, see Pakkanen 2002, 501–502; 2004a, 268–270; on KDE in general, see Silverman 1986, 7–74; on archaeological analysis and KDE, see Baxter/Beardah 1996; Beardah/Baxter 1999; Baxter 2003, 29–37. Histograms are commonly used as input distributions in computer simulations (see e.g. Law/Kelton 2000, 335–337), but the stepped structure may produce inadvertent quantal qualities in the simulated data sets (Pakkanen 2004a, 267). The computer modules used in the cosine quantogram analysis, Monte Carlo simulations, and producing the KDE distributions have been programmed by the author of this paper on top of Survo MM. Cosine quantogram analysis is now a standard feature of Survo MM (this module has been implemented by Seppo Mustonen).

istics of the non-quantal data sets should be<sup>22</sup>. In order to avoid producing the quantal properties of the original data, it is necessary to smooth the KDE curve, and this can be done by manipulating the window- or band-width  $h$  which corresponds to the class-width in histograms<sup>23</sup>: when  $h$  is small, the data structure of the original dimensions can be observed more in detail, and when large, the KDE distribution is very smooth (Fig. 2).

Since the effect of the input distributions on Monte Carlo simulation and cosine quantogram analysis has been questioned by P. R. Freeman<sup>24</sup>, several different KDE distributions with slightly varying band-widths will be used in the following. One thousand simulations are usually regarded sufficient for a statistical test at the 5% level of significance, but I have run three sets of 1,000 simulations for each data model to examine the variation between different Monte Carlo runs<sup>25</sup>. The range for the window-widths used in the Erechtheion inscription dimension simulations is 0.2–0.4 (Fig. 2) and the ‘objective’ values for  $h$  vary between 0.24–0.40<sup>26</sup>.

The results of the Monte Carlo simulations using the different KDE data models are presented in Table 2: no differences can be observed between the simulations using the various band-widths to produce the replica data sets; also, discrepancies between the different simulation runs are rather small. All runs have recognised the higher quantum peak of 3.70 at 0.162 m as significant at the 5% level and rejected the second peak at 0.325 m. The results of the different simulations can be combined to obtain more accurate values based on 9,000 runs (line j in Table 2): the score for 5% significance level can be determined as 3.41 (the dotted line in Fig. 1), and the Erechtheion peak height at the half-foot mark of 0.162 m is topped in only 1.5% of the simulations.

It is highly important that the cosine quantogram analysis recognises a quantum of 0.162 m in the block dimensions and therefore gives strong support for the

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<sup>22</sup> The parallel with bootstrap-techniques is evident (*cf.* Manly 1997, 34), though bootstrapping itself cannot be used to produce replica data sets: since bootstrap is based on the possibility of an observation being replicated in the resampled data set, the method produces emphasised quantum peaks which is exactly the opposite than what the properties of a simulation data set should be; Pakkanen 2002, 502; 2004a, 264–266.

<sup>23</sup> The optimal width of  $h$  in the KDEs can be calculated in several different ways. I have used C. C. Beardah’s MATLAB routines to calculate the optimal window-widths of the KDEs; see Baxter/Beardah 1996, 405–408.

<sup>24</sup> Freeman 1976, 23. Freeman’s Bayesian posterior distributions can be shown to be very closely related to Kendall’s cosine quantogram method; see Silverman 1976, 44–45.

<sup>25</sup> On the number of random data sets, see e.g. Manly 1997, 80–84.

<sup>26</sup> The band-width  $h$  calculated using Solve-The-Equation method (STE) is 0.236, one-, two- and three-stage Direct-Plug-In (DPI) methods 0.345, 0.305 and 0.269 respectively, Smooth-Cross-Validation (SCV) 0.305, and Normal method 0.396. For the methods, see Baxter/Beardah 1996, 397–408.

1. KDE Distribution	2. $\phi(q)$ , $\alpha = 5\%$	3. $\alpha$ , $\phi(q) = 3.70$
a. $h = 0.2$ , 1st run	3.44	1.4%
b. 2nd run	3.36	1.7%
c. 3rd run	3.38	1.2%
d. $h = 0.3$ , 1st run	3.43	2.1%
e. 2nd run	3.45	1.9%
f. 3rd run	3.41	1.1%
g. $h = 0.4$ , 1st run	3.43	1.4%
h. 2nd run	3.39	1.6%
i. 3rd run	3.38	1.5%
j. Combined results of a-i, $n = 9,000$	3.41	1.5%

Table 2. Results of the Monte Carlo simulations ( $n = 1,000$  for each run). The KDE distributions used as simulation data models are based on Table 1, column 3.

existence of a foot-standard of 0.324 m in the Erechtheion. Comparison of the inscription data and the simulation results also confirms that cosine quantogram analysis is a valid way of conducting metrological research: since the inscription was only used to select the blocks subjected to statistical analysis, the results of the initial interpretation of the inscription dimensions (column 4 in Table 1) and the quantogram method are independent from each other. In other words, statistical analysis of the block dimensions would have detected the quantum at 0.162 m as significant at the 5% level even without the information of the block dimensions given in the inscription.

Based on comparison of the inscription and the actual block measurements, the 95% bootstrap confidence interval for the foot-standard length was established above as 0.316–0.327 m. It remains to be seen whether cosine quantogram analysis could be used to determine a more precise range than this. Kendall suggests that the precision with which the size of the quantum is known can be calculated as follows: a mathematical model of the data is first used to create a random sample of dimensions  $X$  and quantal properties are introduced to these values by first calculating the nearest integer  $L$  to  $X/q$  and then replacing each  $X$  by

$$X' = Lq + \sigma e, \quad (3)$$

where  $\sigma$  is standard deviation and  $e$  a standardised Gaussian random variable:  $Lq$  can be defined as a quantal target length disturbed by the error  $\sigma e$ . Kendall also showed that the expectation value for the standard deviation  $\sigma$  can be determined as

$$\sigma = \sqrt{q^2 \times \ln(S/\sqrt{2N}) / -2\pi^2}, \quad (4)$$

where  $S$  is the maximum quantum score; the only restriction is that the number of measurements  $N$  should be large (Kendall 1974, 253–254, 258–260). For the Erechtheion block measurements  $q$  is 0.1619 m and  $S$  3.70, so  $\sigma$  can be calculated as 0.0273 m. Since  $N = 21$  and cannot be classified as large, it is necessary to compare the values of sample standard deviation  $s$  for error  $\varepsilon$  in formula (1) and  $\sigma$ :  $s$  can be calculated as 0.0282 m, which is almost identical with  $\sigma$ . Therefore, the expectation value  $\sigma$  can be used in the simulations.

The new  $X$  values were produced using a KDE distribution with  $h = 0.3$ . Two hundred new sets of simulated  $X^i$ -values were created and analysed using cosine quantogram method: the maximum peaks had a range of 0.1604–0.1640 m and standard deviation of 0.0009 m; the 95% confidence interval for the mean can be calculated as 0.3237–0.3244 m. Therefore, based on cosine quantogram analysis of the block dimensions named in *IG F<sup>3</sup>.474*, the best estimate for the Erechtheion foot-unit can be defined with 95% probability as  $324.0 \pm 0.4$  mm. This is approximately 15 times more precise than the initial comparison of block measurements and inscription data would have indicated, so the benefits of employing the method are quite apparent.

### **Analysis of the Erechtheion plan dimensions**

To test whether the foot-unit defined on the basis of blocks listed in *IG F<sup>3</sup>.474* could also be linked with the general execution of the Erechtheion, it is necessary to study a larger set of building dimensions. In order to avoid selecting only measurements which fit a predetermined pattern, I have chosen to use all dimensions between 0.28–6.00 m given in Stevens' state plan of the Erechtheion (Stevens *et al.* 1927, pl. 2.). The lower limit is selected so as to comprise dimensions close to the one foot mark of any plausible Greek measurement-standard; including dimensions longer than 6 m could slightly distort the analysis of the lower range of the tested quantum lengths: the fit of very long dimensions to small quanta can be quite arbitrary, so I have excluded the principal width and length measurements from the analysis. This does not mean that the main building dimensions could not be expressed in terms of the foot-unit in studies of architectural design, only that the statistical analysis can benefit from their omission.

The 52 Erechtheion plan dimensions within the above defined range are listed in Table 3. The studied range for  $q$  is 0.06–0.40 m, the same as in the inscription data analysis above, and the cosine quantogram plot based on the measurements in Table 3 is presented as Fig. 3. There is a single clear peak at 0.1623 m with a score of 3.532. The results of the Monte Carlo computer simulations based on a large range of different non-quantal KDE distributions are collected in Table 4. No systematic changes can be seen between the runs with varying window-widths used

1. Element	2. Measured dimension (m)	3. Dimension in terms of foot-unit of 0.324 m	4. Discr.
Axial column spacing, E facade	2.114	6 <sup>1</sup> / <sub>2</sub> '	0.008
Distance from stylobate edge to column centre, E facade	0.532	1'10"	0.006
Step width, NE & SE corners of E facade	0.350	1'1'	0.006
Step depth, E facade	0.321	1'	-0.003
Wall block width, E wall	0.638	2'	-0.010
Wall block width, S & N wall	0.692	2'2"	0.003
Distance of anta face to column centre, E facade	1.825	5'10"	0.002
Anta side projection length, E facade	0.333	1'	0.009
Corner block width, E facade	1.630	5'	0.010
Stylobate block depth, E facade	1.320	4'1"	0.004
Second step block width, E facade	2.074	6'6"	0.008
Second step corner block length, E facade	3.590	11'1"	0.006
Second step block length, E facade	1.300	4'	0.004
First step corner block length, E facade	1.334	4'2"	-0.002
First step, length of block next to corner block, E facade	1.624	5'	0.004
Projection of the Karyatid Porch	3.561	11'	-0.003
Width of E opening, Karyatid Porch	1.109	3'7"	-0.005
Width of N opening, Karyatid Porch	1.264	3'14"	0.008
Wall width, Karyatid Porch	0.485	1 <sup>1</sup> / <sub>2</sub> '	-0.001
Width of the Karyatid Porch	5.576	17'3"	0.007
Length of NW wall, Karyatid Porch	1.203	3'11"	0.008
Distance from edge of steps to wall, Karyatid Porch	0.929	2'14"	-0.002
Length of W wall (S stretch)	3.328	10 <sup>1</sup> / <sub>4</sub> '	0.007
Door width, W facade	1.344	4'2"	0.008
Length of W wall (N stretch)	5.165	15'15"	0.001
Block width, W facade	0.675	2'1"	0.007
Krepis block length, W facade	1.276	3'15"	0.000
Projection of N Porch towards W	3.714	11'7"	0.008
Recess width, SW wall of N porch	0.615	1'14"	0.007
Anta width, SW wall of N porch	0.341	1'1"	-0.003
Distance between anta face and column centre, N porch	2.708	8'6"	-0.006
Axial column spacing, sides of N porch	3.067	9'7"	0.009
Distance from stylobate edge to column centre, N porch	0.683	2'2"	-0.005
Step width, N porch	0.328	1'	0.004
Axial column spacing, corner of N porch	3.149	9 <sup>3</sup> / <sub>4</sub> '	-0.010
Axial column spacing, centre of N porch	3.097	9'9"	-0.001

Length of N wall (W stretch)	1.130	3 <sup>1</sup> / <sub>2</sub> '	-0.004
Anta width, N porch	0.799	2'7"	0.009
Anta projection, N porch	0.366	1'2"	0.002
Width of opening, N porch	1.318	4'1"	0.002
Jamb width, N porch	0.513	1'9"	0.007
Door width, N porch	2.427	7 <sup>1</sup> / <sub>2</sub> '	-0.003
Length of N wall (stretch at the SE corner of N porch)	2.547	7'14"	-0.005
Stylobate block depth, N porch	1.380	4 <sup>1</sup> / <sub>4</sub> '	0.003
Pavement block length, N porch	1.318	4'1"	0.002
Pavement block width 1, N porch	1.001	3'1"	0.009
Pavement block width 2, N porch	0.967	3'	-0.005
Pavement block width 3, N porch	1.154	3'9"	-0.000
Pavement block width 4, N porch	0.918	2'13"	0.007
First step block length, N porch	1.547	4 <sup>3</sup> / <sub>4</sub> '	0.008
First step corner block length, N porch	1.373	4 <sup>1</sup> / <sub>4</sub> '	-0.004
Second step block length, N wall	1.300	4'	0.004

Table 3. Erechtheion plan dimensions (based on Stevens et al. 1927, pl. 2).

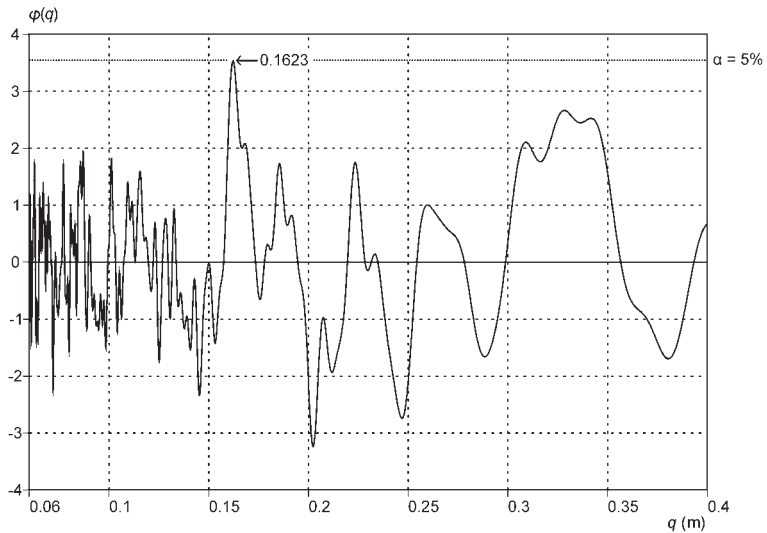


Fig. 3. Cosine quantogram of the Erechtheion plan measurements listed in Table 3.

1. KDE Distribution	2. $\phi(q)$ , $\alpha = 5\%$
a. $h = 0.1$ , 1st run	3.54
b. 2nd run	3.59
c. 3rd run	3.48
d. $h = 0.2$ , 1st run	3.46
e. 2nd run	3.59
f. 3rd run	3.56
g. $h = 0.3$ , 1st run	3.54
h. 2nd run	3.57
i. 3rd run	3.55
j. $h = 0.4$ , 1st run	3.52
k. 2nd run	3.53
l. 3rd run	3.55
m. $h = 0.5$ , 1st run	3.60
n. 2nd run	3.52
o. 3rd run	3.54
p. $h = 0.6$ , 1st run	3.53
q. 2nd run	3.61
r. 3rd run	3.51
s. Combined results of a–r, $n = 18,000$	3.545

Table 4. Results of the Monte Carlo simulations ( $n = 1,000$  for each separate run). The KDE distributions used as simulation data models are based on Table 3, column 2.

to produce the simulation data sets<sup>27</sup>: different runs of 1,000 repetitions produced a range of 3.46–3.61 for the 5% significance level, and the highest quantum score falls within this range. The combined results of the 18,000 simulations place the 5% level of significance at 3.545, so the peak falls short of being statistically significant only by a tiny fraction. When the maximum peak and significance level are this close to each other, it is necessary to run more simulations than usual: 3,000 would have been a sufficient amount to determine the level precisely enough, and importantly all different distributions produce the same result.

The plan dimensions can also be used to highlight the role of data selection in the analysis (*cf.* Fieller 1993, 286). All measurements which Stevens considered significant enough to report in his plan and fell within the defined range of 0.28–6.00 m were included, and omitting even a single badly fitting dimension would have notably changed the height of the calculated quantum score. For example, the stylobate block of the north porch has a depth of 1.380 m, or  $4\frac{1}{4}$  feet of 0.324 m ( $4.25 \times 0.324 \text{ m} = 1.377 \text{ m}$ ); since the detected quantum is equivalent to a half-

<sup>27</sup> The calculated band-widths  $h$  are as follows: STE 0.302, DPI-1 0.390, DPI-2 0.341, DPI-3 0.315, SCV 0.342, and Normal 0.504.



foot, all dimensions which can accurately be expressed in terms of quarter-feet result in the worst possible ‘error’  $\varepsilon$  in formula (1) and produce a notable negative impact on the quantum score: if this one measurement is excluded from the analysis, the height of the maximum peak jumps to 3.76 which would have been instantly recognised as statistically highly significant.

Before proceeding to a study of the plan dimensions expressed in terms of the newly defined foot-standard, it is necessary to investigate how Greek foot-units are typically, and inappropriately, used in architectural studies. Standard metrological analyses which start by taking a set of building dimensions and expressing them in terms of possible measurement units do not in my view advance our understanding of Greek architectural design, and the reason is quite simple: almost any metric dimension can be expressed sufficiently well in terms of at least one of the proposed units and its subdivision into dactyls or finger-breaths. The unit selection and measurement ranges presented in Fig. 4 are based on a recently published representative article (Wilson Jones 2001), and it shows the relationship between the different foot-unit ranges and the ‘grey’ areas between these units: on the left of the figure below 500 mm there are still short stretches which cannot be expressed in terms of the foot-units and their related dactyls, but the awkward areas rapidly disappear towards the right. The enlarged area displays the situation around the one meter mark: there is only one very insignificant grey zone, and even just the ‘Attic’ and ‘Doric’ feet cover nearly the complete spectrum. Stretching the size and number of possibly used foot units and introducing units as small as half-dactyls render most metrological analyses empty exercises.

Even though the results of the analysis of the Erechtheion plan dimensions fall a little short of being statistically valid, it is very encouraging that the most likely quantum detected in the plan analysis is the same as the one based on the inscription blocks. Therefore, it is at this point justifiable to proceed to scrutinise how well the plan measurements of the Erechtheion can be expressed in terms of the defined foot-unit: this must, however, follow a quantitative analysis of the dimensions in question.

Only 14 of the 52 dimensions, or a little more than a quarter, are expressible in terms of a multiple of the determined quantum of 0.162 m (marked with bold typeface in column 3 of Table 3). The concentration of measurements around the half- and full-foot marks is still sufficient to produce an almost statistically significant maximum peak. The dactyl part of the dimension expressed in terms of the foot-unit of 0.324 m can be used to illustrate the degree of clustering around the quantum. Fig. 5 presents a histogram of the dactyl values in column 3 of Table 3: there is visible grouping especially around zero and also eight dactyls, though less in the latter case. The histogram illustrates well the general robustness of the cosine quantogram method: despite the spread of the dactyl values across the full range, an appropriate quantitative method can deal with the significant amount of

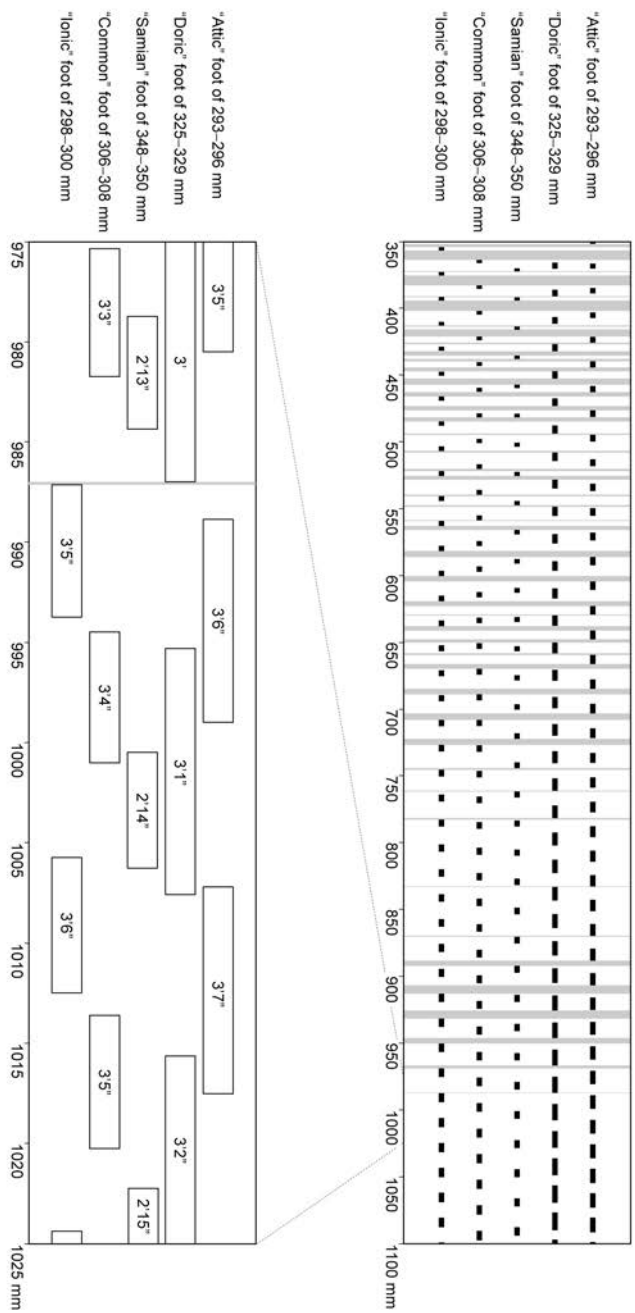


Fig. 4. Relationship of ‘standard’ Greek foot-units and the ‘grey’ areas between dimensions expressed in terms of these units.

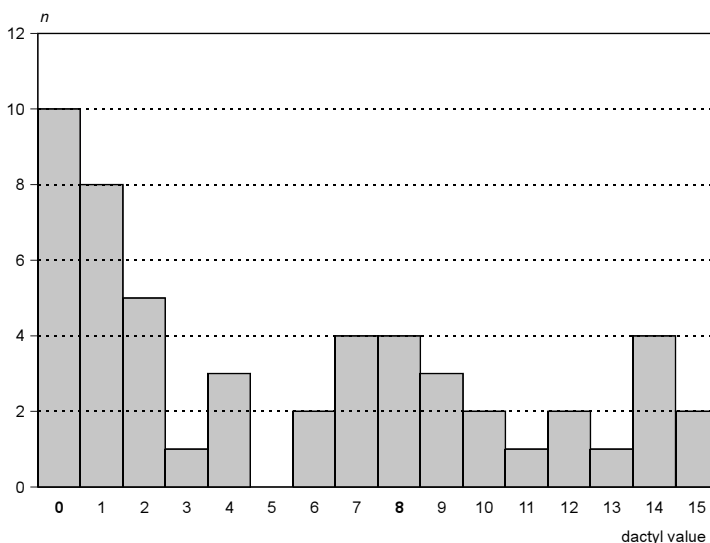


Fig. 5. Histogram of the dactyl values of the dimensions in Table 3, column 3.

noise produced by the Greek design and construction practices as long as there is some tendency to use discreet steps in the sizes of the building elements. In this case the mean of the errors  $\varepsilon$  in formula (1) can be calculated as 29 mm; therefore, it can be estimated that with a sufficient number of measurements and the length of the detected quantum in the region of half a foot, cosine quantogram analysis can cope with discrepancies of at least in the region of  $\pm 25$  mm, so greater than plus or minus one dactyl. The traditional approach to Greek metrology relies on the errors being fractions of a dactyl, so the results produced by these analyses cannot be considered reliable.

At this point it is necessary to turn back to Bankel's metrological scale analysis of the Erechtheion-foot. In light of the plan analysis presented above it is clear that Bankel's method cannot succeed in determining the foot-unit correctly: the executed dimensions fluctuate too much for his method to produce a meaningful result using subdivisions of the foot as small as a dactyl. For example, his suggestion that the stylobate width of the Erechtheion North Porch, 10.717 m (Dinsmoor 1950, 340; see also Büsing 1985, 159–160), can be defined as 525 dactyls (Bankel 1983, 89–91) is based solely on the predetermined assumption of the foot-unit length. In terms of the ancient measurement-unit used in the building, this particular dimension can now be more correctly put forward as 529

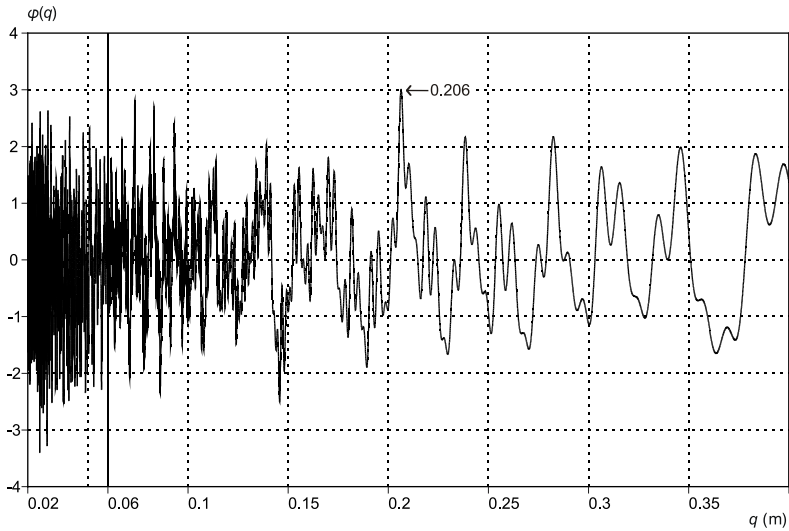


Fig. 6. Cosine quantogram based on Bankel’s dimensions of the Erechtheion North Porch.

dactyls<sup>28</sup>. Even though Bankel’s data set, only nine measurements, is too small for reliable statistical analysis<sup>29</sup>, a cosine quantogram can be used to pinpoint what the particular shortcomings of the analysis based on metrological scale are in this case (Fig. 6). The curve is very erratic to the left of 0.1 m, and this due to the small number of dimensions. The graph starts at 0.02 m, well below the actual detection limit of 0.06 m for the quantum (solid line in the graph): no significant peak emerges at the lower end, so Bankel’s suggested unit of 0.32674 m is not supported by his own data even in the dactyl range<sup>30</sup>. The highest peak emerges at 0.206 m, and even though several different interpretations of a single value can always be proposed, none of them have any statistical significance in this case due to the small data set: for example, 0.2063 m could be 10 dactyls of a foot of 0.330 m, 11 dactyls of 0.300 m or even two thirds of 0.309 m.

Another branch of standard metrology starts with an assumption that the major dimensions of the buildings should be possible to express in round numbers of

<sup>28</sup> 10.717 m expressed as feet of 0.324 m is 33’1” (discrepancy 0.5 cm). Bankel’s method also fails to produce a meaningful result also in the case of two fourth-century temples I have studied; for Tegea, see Pakkanen 2005, and for Stratos, Pakkanen 2004b, 111–119, esp. note 58.

<sup>29</sup> For a small sample such as this it is not e.g. sensible to calculate significance levels using Monte Carlo simulations.

<sup>30</sup> 0.32674 m / 16 ≈ 0.0204 m, so Bankel’s dactyl length is at the very beginning of the graph.

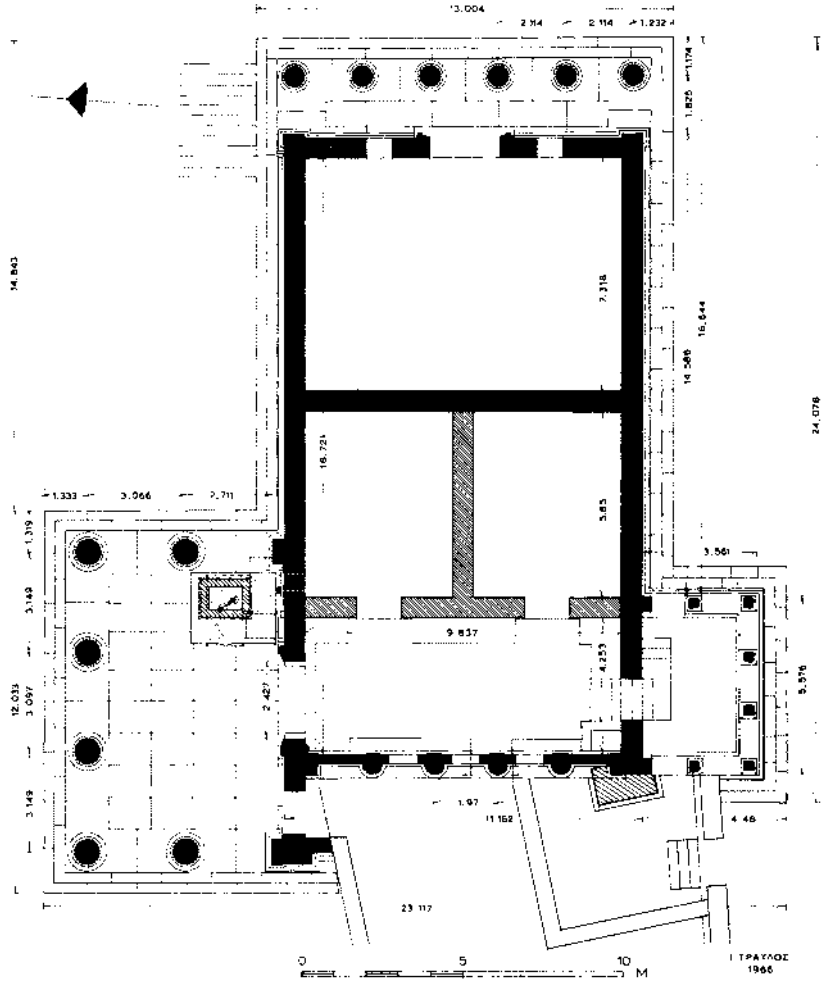


Fig. 7. Plan of the Erechtheion with principal dimensions (Travlos 1971, fig. 280).

feet. The choice of the dimensions inevitably determines the outcome of these analyses, and in order to avoid intentionally selecting well or badly fitting measurements, I have taken the principal building dimensions of the Erechtheion as they are reported by John Travlos (Fig. 7: Travlos 1971, fig. 280). The results of expressing the measurements in terms of the foot-units in the range 0.324–0.328 m are rather disastrous for the initial hypothesis that round numbers of feet can be detected in the building plan (Table 5): the unit lengths of 0.325 and 0.328 m

Table 5.  
Principal dimensions of the Erechtheion plan expressed in terms of different lengths of the foot-unit (based on Travlos 1927, fig. 280).

1. Element	2. Measured dimension (m)	3. F = 0.324	4. Discr.	5. F = 0.325	6. Discr.	7. F = 0.326	8. Discr.	9. F = 0.327	10. Discr.	11. F = 0.328	12. Discr.
Width at first step, E facade	13.004	40'2"	0.003	40'	0.004	39'14"	0.005	39'12"	0.006	39'10"	0.007
Length at first step, S side	24.078	74'5"	0.001	74'1"	0.008	73'14"	-0.005	73'10"	0.003	73'7"	-0.010
Length from E facade to Karyatid Porch	16.644	51'6"	-0.002	51'3"	0.008	51'1"	-0.002	50'14"	0.008	50'12"	-0.002
Length of S wall to Karyatid Porch	14.586	45'	0.006	44'14"	0.002	44'12"	-0.003	44'10"	-0.006	44'8"	-0.010
Width of W facade	11.162	34'7"	0.004	34'6"	-0.010	34'4"	-0.003	34'2"	0.003	34'	0.010
Width at first step, W facade	23.117	71'6"	-0.009	71'2"	0.001	70'15"	-0.009	70'11"	0.002	70'8"	-0.007
Length at first step, N porch	12.033	37'2"	0.005	37'	0.008	36'15"	-0.009	36'13"	-0.005	36'11"	-0.001
Length from E facade to N porch at first step	14.843	45'13"	0.000	45'11"	-0.005	45'8"	0.010	45'6"	0.005	45'4"	0.001
Internal length	18.721	57'12"	0.010	57'10"	-0.007	57'7"	-0.004	57'4"	0.000	57'1"	0.005
Internal width	9.837	30'6"	-0.005	30'4"	0.006	30'3"	-0.004	30'1"	0.007	30'	-0.003
Internal length of E room	7.318	22'9"	0.008	22'8"	0.005	22'7"	0.003	22'6"	0.001	22'5"	-0.001

produce two ‘hits’<sup>31</sup>, but in general no significant pattern can be observed. Even though the inscription demonstrates a tendency to use discrete multiples of half-feet in the blocks, this does not necessarily translate into exact multiples of the quantum in the major building dimensions: fifth-century Attic Ionic design is, at least in the case of the Erechtheion, still a long step away from the transparency displayed by the Late-Classical and Hellenistic Ionic temples with their strictly modular design (see e.g. Coulton 1977, 70–71; Wilson Jones 2001, 675–676).

J. J. Coulton’s suggestion that Greek architects probably used successive systems of proportion (Coulton 1975, 68–73) could well be the reason why it is so difficult to express the modern measurements in terms of a coherent ancient unit. Starting from Vitruvius’ definitions for the Doric and Ionic orders, Coulton elegantly demonstrates the differences between modular and proportional systems. Vitruvius’ Doric has a radial pattern with most of the dimensions derived fairly directly from the module, while the Ionic is much more linear (Vitr. 4.3.3–10, 3.5.1–15). Following Vitruvius’ rules for the Doric produces a transparent design, and establishing the relationships between the various parts of the building should be a quite simple task involving not much more than testing the different possibilities with a calculator. Analyzing a successive system such as Vitruvius’ Ionic can be a much more difficult exercise. If the dimensions are rounded at each step, it is possible that the sizes of the elements higher up in the facade bear no precise proportional relationship to the size of the initial module: the whole design is likely to be far more opaque than the Doric and cannot be considered anymore based on a ‘real’ modular system.

### Conclusions

The Erechtheion block inventory inscription is an invaluable guideline in the analysis of the length of the foot-standard employed in the construction of the temple. In this paper it is demonstrated that cosine quantogram method can provide a robust tool for detecting the unit lying under the measurement data: using only blocks identified in the inscription, the length of the Erechtheion foot-unit can be defined with 95% probability as  $324.0 \pm 0.4$  mm; the results of the block analysis are further supported by a study of 52 plan dimensions of the building. A change of a few millimetres compared to previous suggestions for the unit length may not seem that significant, but even in the medium range building dimensions there is a notable difference: for example, the projection of the Karyatid Porch of 3.561 m can now be recognised as 11 feet and not 10 feet and 10 dactyls<sup>32</sup>.

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<sup>31</sup> 0.325 m: width of the east facade and the north porch; 0.328 m: width of the west facade and internal width.

<sup>32</sup>  $3.561 \text{ m} / (0.324 \text{ m} / 16) \approx 175.9'' \approx 11'$ ;  $3.561 \text{ m} / (0.327 \text{ m} / 16) \approx 174.2'' \approx 10' 10''$ .

Based on the Erechtheion it can be suggested that using block measurements rather than principal building dimensions, such as the total width and length of the structure, will more probably result in the discovery of a statistically valid architectural foot-standard. Also, provided that a sufficient number of blocks were executed reasonably precisely in multiples of half-feet, a unit of that magnitude can be detected even if there are discrepancies of up to  $\pm 25$  mm in the execution of the building elements. The three key issues in determining the lengths of foot-units on the basis of building dimensions are as follows: 1) the use of an appropriate analysis method, 2) data selection, and 3) the number of analysed measurements. It is vital that the selection of dimensions can be shown to be systematic and that even though small well selected samples can produce a significant result, such as here by using the inscription as a guide, larger data sets are more likely to produce statistically valid results.

The conclusions reached in this paper should have a wider impact than just related to Athenian fifth-century architecture. Despite the building block dimensions listed in *IG F.474.8–155*, studies on the Erechtheion foot-unit have failed to correctly identify its length. This illustrates the serious shortcomings of the standard metrological approach in being able to derive Greek foot-standards from building dimensions. Since there is little external evidence on the units<sup>33</sup>, statistical analysis can provide a fresh start. No results of the previous metrological studies should be taken as granted, and only a thorough re-examination of all available evidence employing a proper quantitative method can provide a stable ground on which further analyses of Greek architectural measurement-units can be built. The paper also highlights the need to turn again away from simplistic metrological calculations and analyses towards looking building design as a whole and the role of proportional systems. All studies of building measurements should seriously take into consideration the possibility that the Greek architects used a successive system of proportion as their principal design tool, thus producing intriguing buildings which easily escape the attempts of modern scholars to fully grasp their basic design principles.

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<sup>33</sup> For example, the dimensions of the Salamis relief can only be used to argue that the length of one of the carved units is 322–328 mm, depending on how it is measured; see Dekoulakou-Sideris 1990 and Wilson Jones 2000, 77–81.



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